ANALYSIS OF MATHEMATICAL REPRESENTATION, COMMUNICATION AND CONNECTION IN TRIGONOMETRY

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Abstract

Trigonometry is one of mathematics branch that is difficult to understand according to some students. Some students still struggle to solve mathematical problems associated with trigonometry. In this research, the researchers tried to analyze students' mathematical representation, mathematical communication and mathematical connection in answering mathematics problems. This research is done to obtain data in case recognizing the students' ability of representation, communication, and connection in solving mathematics problems, particularly in trigonometry. These three capabilities are chosen because they are the key of skills in solving mathematical problems. This research is expected to detect students' difficulties in solving problems on trigonometric material accordingly to provide appropriate treatment to students.

The research method used is a qualitative description by conducting qualitative analysis on students' work result and interview to obtain more information. From the analysis results obtained it can be concluded that most of the students still have problems in representing the problem and in building connections with the materials that have been studied. From this result, it can be concluded that the students still need guidance in mathematical representation and connection through the learning process in order to improve their ability in solving the mathematics problem, particularly on trigonometry.

Keywords: trigonometry, representation, communication, connection.

Introduction

Mathematics is one of the sciences that must be studied by the students. Many students find mathematics as a difficult subject to understand. Most of them have negative views beforehand that they will not be able to understand math any more when it comes to formulas that look complicated. Based on the research that has been done by previous authors obtained data that trigonometry is the most difficult material to understand because the students consider it too abstract. Actually, these difficulties can be minimized by improving low math skills. The ability
to learn mathematics itself is divided into 5 parts (NCTM) that is problem solving ability, reasoning ability, communication ability, connection making ability, and representation ability. In this article will only be discussed more deeply related to the three abilities in learning mathematics namely: mathematical representation, mathematical communication and mathematical connections. With this research, researchers can understand the weakness of mathematics skills of each student so that it can immediately overcome in order to get the maximum learning for the students.

Theory

A. Theory of Mathematical Representation

Representation ability is a way that students use to communicate ideas, or answers the problem (Nurhayati, 2013). In order to communicate something (e.g. a math problem) a student needs drawings, graphs, diagrams and other representational forms. It is expected that with representation, the problem that initially looks difficult to solve becomes easier to solve because the problem can be presented more simply. Mathematical representation is very important because it can help students in organizing their thoughts when solving problems. In NCTM it is stated that representation is the center for learning mathematics. Students can develop and deepen their understanding of the mathematical concepts and relationships they make, compare, and use varied representations.

Many mathematicians categorize mathematical representations which as a whole have almost the same classification. In this article we are guided by two expert sources in the categorization of the mathematical representation capabilities, they are Jerome Bruner with
the Learning Theory as well as the grouping of mathematical representations according to Hiebert and Carpenter.

1. Jerome Bruner

   a. Enactive Representation (enactive)

      At this stage the child learns knowledge where knowledge is actively learned by using concrete objects or using real situations.

   b. Iconic Representation (iconic)

      In this type, the knowledge is represented in the form of visual imagery, drawings, or diagrams depicting concrete activities or concrete situations found in the enactive stage.

   c. Symbolic Representation

      At this symbolic stage, learning is represented in the form of abstract symbols, the arbitral symbols used by agreement in the field concerned, both verbal symbols (e.g. letters, words, sentences), mathematical symbols and other abstract symbols.

2. Hiebert and Carpenter

   a. Internal Representation

      Internal representation is closely related to the process of recovering the knowledge that has been acquired and stored in the memory as well as the relevance of the need to be used when necessary. The process of internal representation is certainly not observable and can not be assessed directly because it is a mental activity in one's mind.
b. External Representation

The results of this embodiment can be expressed either orally, written in the form of words, symbols, expressions, or mathematical notations, drawings, graphs, diagrams, tables, or physical objects in the form of props.

B. Mathematical Communication Theory

Within (1992) states that communication skills become important when discussion between students is done, where students are expected to be able to express, explain, describe, hear, ask and cooperate so as to bring students to a deep understanding of mathematics. Indicator of students' mathematical communication ability according to Ross (in Al Jupri, 2007) in the form of written communication is as follows:

- Describe the problem situation and state the problem solution using images, tables, charts, algebraically.
- Declare results in written form.
- Using a thorough representation to express a mathematical concept and its solution.
- Create a mathematical situation by providing ideas and information in written form.
- Use the language and mathematical symbols appropriately.

Indicators of spoken communication in the form of discussion are students can:

- Take an opinion on the issues discussed.
- Participate actively in responding to the opinions of other students.
Want to ask questions if there is something that cannot be understood.

Listen seriously when other students express an opinion so they can understand the opinion.

C. Mathematical Connection Theory

The students' mathematical connection ability is related to connecting the prior knowledge that students have with new knowledge so that the material can have profound meaning. Supporting activities in improving students' mathematical connections are when students look for linkages between mathematical topics, and look for connection between external contexts outside mathematics and pure mathematics.

Three aspects of mathematical connection ability, namely:

- Write down the problems of everyday life in the form of mathematical models.
  
  In this aspect, students are expected to be able to connect between problems in everyday life and math.

- Write down the mathematical concept that underlies the answer.
  
  In this aspect, students are expected to be able to write the underlying mathematical concepts of answers to understand the interrelationships between the mathematical concepts to be used.

- Write the relationship between objects and mathematical concepts. In this aspect, students are expected to be able to write the relationship between mathematical concepts used in answering the given problem.
Methodology

This research type is qualitative description research that is analyzing student answer result when doing problem in trigonometric material. Place of research is conducted at SMA Pangudi Luhur Yogyakarta on the day Thursday to Saturday of March, 14th – 16th. Object in this research is result of student answer in solving problem related to trigonometric material to know ability of representation, communication and mathematical connection of student. While the subject for this research is 5 students of class X SMA Pangudi Luhur Yogyakarta. In this study, the type of data needed is the result of the analysis of student answers in solving the trigonometric problems seen from the ability of representation, communication, and mathematical connection of students. For data collection methods, the researchers make good observations of learning in the classroom as well as observation of student activities. Second, the researchers provide a written test in the form of a matter of trigonometric material for class X. The last method of data collection is to conduct interviews to further deepen the students’ intent in solving the problem. Instrument of data collection in this research is in the form of trigonometric and student interview. As for method of data analysis, researchers choose technique of data analysis from Miles and Huberman (1984) that is in the form of data reduction, data presentation and conclusion. Here are the details of the research steps:

The first step is to provide problems related to trigonometric material where in the problem contains the ability of representation, communication and mathematical connections to get the solution. This step is used to determine the problem of mathematical ability experienced by students so that later can be used by researchers to overcome the difficulties of students in learning mathematics specifically trigonometric material. The second step is that students working on the problem with their own thinking ability. Students are given the freedom to find
solutions in accordance with their knowledge. The third step is to analyze the student work. The analysis done is to notice at the stages described by the students in finding the solution of each problem. The last step taken is interviewing some students to clarify the answers they have written to avoid misinterpretations.

Results and Discussion

Student 1

Student 1 used iconic representation, seen where student poured the matter into triangular image form. Student also represented the story with a symbolic matter where changing position and distance with the letters then perform calculations with the symbols that have been made and get the right solution. If viewed from an external representation, the student 1 well represented the problem visually visible from the geometry drawing it creates. Students also performed the representation of equations and mathematical expressions in solving the problem that is when doing calculations by utilizing sinus rules. Representation in the form of words is also poured though only slightly.

In written communication, student 1 communicated it in the form of images and symbols and is able to use it to solve the problem appropriately. In connection capabilities, student was able to use materials that have been previously learned as well as using properly and precisely mathematical computing to solve everyday problems.
Student 2

Student 2 tried to represent the problem symbolically with visible marks and letters of the right sign and letters that are expressed in mathematical expression even though the execution of the calculation is not yet accurate because of the algebraic problem. In written communication, student has been able to provide information from the problem on the image with precision. But in doing the calculations, there are still errors and through oral communication, student realized the inaccuracy in the completion of algebra, and from this communication can be explained the misconception in understanding the problem. Student tried to connect by connecting pythagoras theory to solve the given problem. However, due to errors in the representation then the connection that is trying to build also experienced errors.
Student 3

Student 3 tried to use symbolic representation by replacing the sides of the yard with letters. For representation in the form of mathematical expression equation is correct. This is the evidence from the results of calculations that have obtained the correct solution.

In mathematical communication, student could declare problem solutions by providing precise information on images reinforced by oral communication whereby student can clearly explain each step of the settlement they take. In the ability of connections student was able to connect the knowledge that has previously owned the rules of sinus, cosine rules and the area of triangle to solve the story presented. Connection error associated with representation.
Student 4

Student tried to use a symbol representation in which the student puts sides and corners on the yard with letters. Then with the symbols that have been selected, this student used it to perform calculations and got the results of the calculation. From an external representation, student is able to represent problem solving in mathematical expression correctly and perform calculations of the mathematical expression to get the right results.

In written communication, the student can state the solution of the problem by giving the exact description of the image. Student also has used the algebraic way of solving a and b problems correctly. In oral communication with further interviews, student is able to interpret and evaluate ideas, symbols, terms, and mathematical information as well.

Based on the results of student work is known that student is able to provide mathematical models into the form of mathematical equations that use algebraic equations correctly. The
previous relationship of mathematical concepts can be related so that it is a unity to solve math problems properly and correctly in searching the BC side of the students to relate the settlement with the sinus rules and fractional rules. Whereas in searching the total area, the student associates the area of triangle previously learned and the use of triangle using sinus based on the things that have been known.

**Figure 5. Results of student work 5**

Student represents the problem symbolically, where student tries to change the names of the sides of each yard with letters to make it easier to calculate. Student also performs a mathematical expression representation where student solves problems by doing calculations through trigonometric comparison equations even though the student has not been able to get a correct solution. This is because student still lacks the understanding of the use of comparison of trigonometry in right triangle.
In terms of mathematical communication, student tried to pour the idea by using the representation of symbols to change the description of the image. The calculation steps that are trying to build still encounter error, this is because student has not understood the comparison of trigonometry. The symbol used is still not appropriate in calculation.

In oral communication through interviews, it is acquired that student still does not understand the calculation so that there are calculations that are still come from cheating results from friends without knowing the origin of the results. In mathematical connections with relation between mathematical concepts, student has tried to relate the problem presented with previous knowledge that is owned by the comparison of trigonometry in right triangle, but since they do not understand it the students tend not to use it in daily calculation problem.

**Conclusion**

Based on research conducted for 5 students in SMA Pangudi Luhur Yogyakarta, we can notice that:

1. Most of student did symbolic representation correctly, in other hand external representation and equation representation still needed to be enhanced.
2. In communication, some students did not communicate well because they missed the correct representation.
3. In connection, most of students can connect their materials with prior knowledge they had but mistake happen since wrong representation.

From the explanation mentioned, researchers preserve that representation, communication and connection is related each other. Over all, most of the students still have problems in representing the problem and in building connections with the materials that have been studied.
Researchers can conclude that the students still need guidance in mathematical representation and connection through the learning process in order to improve their ability in solving the mathematics problem, particularly on trigonometry.

References


